

2024 秋季初二数学每日一题打卡 005

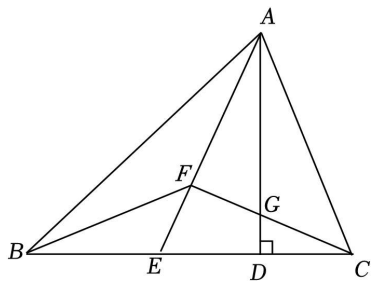
005 试题来源:2023 年秋季姑苏区月考

如图,在 $\triangle ABC$ 中, AD 为 BC 边上的高, AE 是 $\angle BAD$ 的角平分线,点 F 为 AE 上一点,连接 BF , $\angle BFE = 45^\circ$.

(1) 求证: BF 平分 $\angle ABE$;

(2) 连接 CF 交 AD 于点 G ,若 $S_{\triangle ABF} = S_{\triangle CBF}$,求证: $\angle AFC = 90^\circ$;

(3) 在 (2) 的条件下,当 $BE = 3$, $AG = 4.5$ 时,求线段 AB 的长.



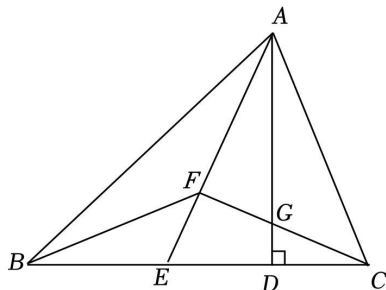
试题解析:

如图,在 $\triangle ABC$ 中, AD 为 BC 边上的高, AE 是 $\angle BAD$ 的角平分线,点 F 为 AE 上一点,连接 BF , $\angle BFE = 45^\circ$.

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【解答】(1) 证明: $\because AE$ 是 $\angle BAD$ 的角平分线,

$$\therefore \angle BAD = 2\angle BAF,$$

$$\because \angle BFE = 45^\circ, \therefore \angle FBA + \angle BAF = 45^\circ,$$

$$\therefore 2\angle FBA + 2\angle BAF = 90^\circ,$$

$\because AD$ 为 BC 边上的高,

$$\therefore \angle EBF + \angle FBA + 2\angle BAF = 90^\circ,$$

$$\therefore 2\angle FBA = \angle EBA + \angle FBA,$$

$$\therefore \angle EBF = \angle FBA, \therefore BF \text{ 平分 } \angle ABE;$$

(2) 证明:过点 F 作 $FM \perp BC$ 于点 M , $FN \perp AB$ 于点 N ,

$$\because BF \text{ 平分 } \angle ABE, FM \perp BC, FN \perp AB, \therefore FM = FN,$$

$$\because S_{\triangle ABF} = S_{\triangle CBF}, \text{ 即 } \frac{1}{2}AB \cdot FN = \frac{1}{2}BC \cdot FM, \therefore AB = BC,$$

$$\text{在 } \triangle ABF \text{ 和 } \triangle CBF \text{ 中, } \begin{cases} BA = BC \\ \angle ABF = \angle CBF \\ BF = BF \end{cases}$$

$$\therefore \triangle ABF \cong \triangle CBF (SAS), \therefore \angle AFB = \angle CFB,$$

$$\because \angle BFE = 45^\circ \therefore \angle AFB = 135^\circ, \therefore \angle CFB = 135^\circ,$$

$$\therefore \angle CFE = \angle CFB - \angle BFE = 135^\circ - 45^\circ = 90^\circ, \therefore \angle AFC = 90^\circ;$$

(3) 解: $\because \triangle ABF \cong \triangle CBF, \therefore AF = FC,$

$$\because \angle AFC = \angle ADC = 90^\circ, \angle AGF = \angle CGD, \therefore \angle FAG = \angle FCE,$$

$$\text{在 } \triangle AFG \text{ 和 } \triangle CFE \text{ 中, } \begin{cases} \angle AFG = \angle CFE \\ AF = CF \\ \angle FAG = \angle FCE \end{cases},$$

$$\therefore \triangle AFG \cong \triangle CFE (ASA), \therefore AG = EC = 4.5,$$

$$\because BE = 3, \therefore BC = BE + EC = 7.5, \because \triangle ABF \cong \triangle CBF, \therefore AB = BC = 7.5.$$

【点评】本题考查了角平分线的性质:角的平分线上的点到角的两边的距离相等.也考查了全等三角形的判定与性质.

